What is Calculus?

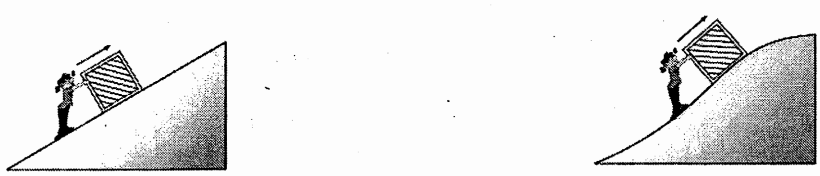
As students when people first begin to study calculus in high school or college, many may be unsure about what calculus is. What are the fundamental concepts underlying calculus? Who contributed for the discovery of calculus and how calculus is used today will be discussed in this paper.

Calculus was invented as a tool for solving problems. Prior to the development of calculus, there were a variety of different problems that could not be addressed using the mathematics that was available. For example, scientists did not know how to measure the speed of an object when that speed was changing over time. Also, a more effective method was desired for finding the area of a region that did not have straight edges. Geometry, algebra, and trigonometry, which were well understood, did not provide the necessary tools to adequately address these problems.

At the time in which calculus was developed, automobiles had not been invented. However, automobiles are an example of how calculus may be used to describe motion. When the driver pushes the accelerator of a car, the speed of the car increases. The rate at which the car is moving or the velocity increases with respect to time. When the driver steps on the brakes, the speed of the car decreases. The velocity decreases with respect to time.

As a driver continues to press on the accelerator of a car, the velocity of that car continues to increase. "Acceleration" is a concept that is used to describe how velocity changes over time. Velocity and acceleration are measured using a fundamental concept of calculus that is called the derivative.

For example, when a man pushes a crate with an unchanging force the crate goes up the incline at an unchanging speed. With some simple physics, formulas and regular mathematics (including algebra and trigonometric), you can compute how many calories of energy are required to push the crate up the incline. Note that the amount of energy expended in each second remains the same.



Regular mathematical problem — Calculus problem

Figure 1: Illustrating the difference in mathematical and calculus problems

For the curving incline, things are constantly changing. The steepness of the incline is changing and the change is not just in increments. For example, its steepness for the first few feet is different from the steepness in the next few feet. It's constantly changing. When the man pushes with a constantly changing force, steeper the incline, the push becomes harder. As a result, the amount of energy expended is also changing, not in every second or every thousandth of a second, but constantly changing from one moment to the next. That's what makes it a derivative problem.

Further, more derivatives can be used to describe the motion of many different objects. For example, derivatives have been used to describe the orbits of the planets and the descent of space shuttles. Derivatives are also used in a variety of different fields. Electrical engineers use derivatives to describe the change in current within an electric circuit. Economists use derivatives to describe the profits and losses of a given business.

The concept of a derivative is also useful for finding a tangent line to a given curve at a specific point. A tangent line is a straight line that touches a curve at only one point when restricted to a very small region. An example of a tangent line to a curve is shown in Figure 2. The straight line and the curve touch at only one point. The straight line is the tangent line to the curve at that point.

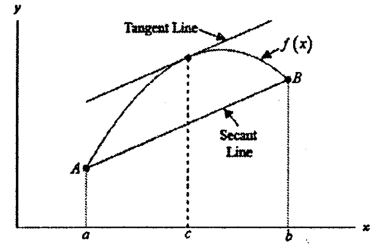


Figure 2: An example of a tangent line to a curve

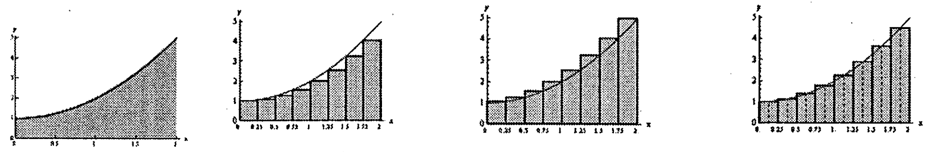
Tangent lines are useful tools for understanding the angle at which light passes through a lens. Derivatives and tangent lines were useful tools in the development and continued improvement of the telescope. Derivatives are also used today by optometrists or eye doctors to develop more effective methods for correcting vision. Physicists use tangent lines to describe the direction in which an object is travelling, and chemists use tangent lines to predict the outcomes of chemical reactions. These are only a few examples of the many uses of tangent lines in science, engineering, and medicine.

Derivatives along with the concept of a tangent line can be used to find the maximum or minimum value for a given situation. For example, a business person may wish to determine how to maximize profit and minimize expense. Astronomers also use derivatives and the concept of a tangent line to find the maximum or minimum distance of earth from the sun.

The derivative is closely related to another important concept in calculus, the integral. The integral, much like the derivative, has many applications. For example, physicists use the integral to describe the compression of a spring. Engineers use the integral to find the "center of mass" or the point at which an object balances. Mathematicians use the integral to find the areas of surfaces, the lengths of curves, and the volumes of solids.

The basic concepts that underlie the integral can be described using two other mathematical concepts that are important to the study of calculus "area" and "limit." Many students know that finding the area of a rectangle requires multiplying the base of the rectangle by the height of the rectangle. Finding the area of a shape that does not have all straight edges is more difficult.

The area between the curve and the x - axis is colored in the figure (a). One way to estimate the area of the portion of the figure that is colored is to divide the region into rectangles as shown in (b) and (c). (b) contains less area than in the colored region and (c) contain more area than in the colored region. To estimate the area of the colored region, the area of the eight rectangles can be added together.



(a) (b) © (d)

Figure 3: Estimating the area of a shape with no straight edges

If a better estimate is desired, the colored region can be divided into more rectangles with smaller bases, as shown in (d). The areas of these rectangles can then be added together to acquire a better approximation to the area of the colored region.

If an even better estimate of the colored region is desired, it can be divided into even more rectangles with smaller bases. This process of dividing the colored region into smaller and smaller rectangles can be continued. Eventually, the bases of the rectangles are so small that the lengths of these bases are getting close to zero.

The concept of allowing the bases of the rectangles to approach zero is based on the limit concept. The integral is a mathematically defined function that uses the limit concept to find the exact area beneath a curve by dividing the region into successively smaller rectangles and adding the areas of these rectangles. By extending the process described here to the study of three-dimensional objects, it becomes clear that the integral is also a useful tool for determining the volume of a three-dimensional object that does not have all straight edges.

An interesting relationship in calculus is that the derivative and the integral are inverse processes. Much like subtraction reverses addition; differentiation (finding the derivative) reverses integration. The reverse of this statement, integration reverses differentiation, is also true. This relationship between derivatives and integrals is referred to as the "Fundamental Theorem of Calculus." The Fundamental Theorem of Calculus allows integrals to be used in motion problems and derivatives to be used in area problems.

Who Invented Calculus?

Pinpointing who invented calculus is a difficult task. The current content that comprises calculus has been the result of the efforts of numerous scientists. These scientists have come from a variety of different scientific backgrounds and represent many nations and both genders. History, however, typically recognizes the contributions of two scientists as having laid the foundations for modern calculus: Gottfried Wilhelm Leibniz (1646–1716) and Sir Isaac Newton (1642–1727).

Leibniz was born in Leipzig, Germany, and had a PhD in law from the University of Altdorf. He had no formal training in mathematics. Leibniz taught himself mathematics by reading papers and journals. Newton was born in Woolsthorpe, England. He received his master's degree in mathematics from the University of Cambridge.

The question of who invented calculus was debated throughout Leibniz's and Newton's lives. Most scientists on the continent of Europe credited Leibniz as the inventor of calculus, whereas most scientists in England credited Newton as the inventor of calculus. History suggests that both these men independently discovered the Fundamental Theorem of Calculus, which describes the relationship between derivatives and integrals.

The contributions of Leibniz and Newton have often been separated based on their area of concentration. Leibniz was primarily interested in examining methods for finding the area beneath a curve and extending these methods to the examination of volumes. This led him to detailed investigations of the integral concept.

Leibniz is also credited for creating a notation for the integral, +". The integral symbol looks like an elongated "S." Due to the fact that finding the area under a curve requires "summing" rectangles, Leibniz used the integral sign to indicate the summing process. Leibniz is also credited for developing a notation for finding a derivative. This notation is of the form. Both these symbols are still used in calculus today.

Newton was interested in the study of "fluxions." Fluxions refer to methods that are used to describe how things change over time. As discussed earlier, the motion of an object often changes over time and can be described using derivatives. Today, the study of fluxions is referred to as the study of calculus. Newton is also credited with finding many different applications of calculus to the physical world.

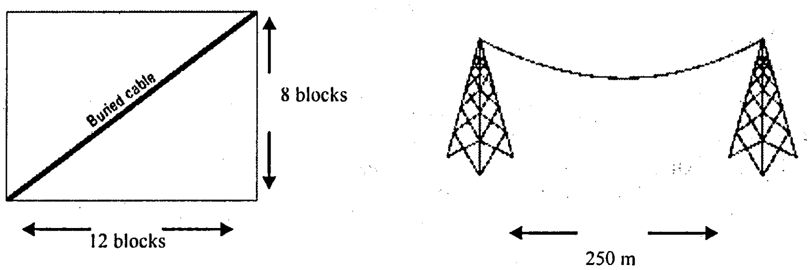
It is important to note that the ideas of Leibniz and Newton had built upon the ideas of many other scientists, including Kepler, Galileo, Cavalieri, Fermat, Descartes, Torricelli, Barrow, Gregory, and Huygens. Also, calculus continued to advance due to the efforts of the scientists who followed.

These individuals included the Bernoulli brothers, L'Hôpital, Euler, Lagrange, Cauchy, Cantor, and Peano. In fact, the current formulation of the limit concept is credited to Louis Cauchy. Cauchy's definition of the limit concept appeared in a textbook in 1821, almost 100 years after the deaths of Leibniz and Newton.

Some Real-World Examples of Calculus

Regular mathematics can solve the straight incline problem.
Calculus can solve the curving incline problem. Here are some examples.

Regular mathematics can determine the length of a buried cable that runs diagonally from one corner of a park to the other. Calculus can determine the length of a cable hung between two towers that has the shape of a catenary (which is different, by the way, from a simple circular arc or a parabola). Knowing the exact length is of obvious importance to a power company planning hundreds of miles of new electric cables. See Figure 4.

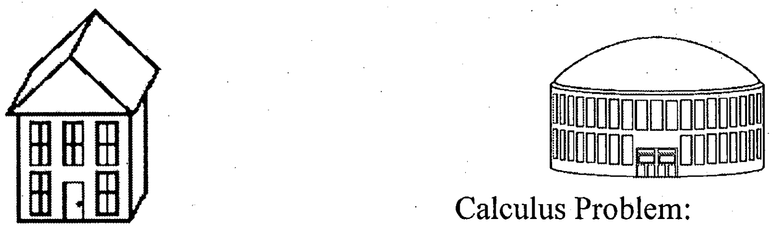


Regular Mathematical problem
How long is the cable?

Calculus problem:
How long is the cable?

Figure 4: Real world examples showing the use of calculus

The area of a flat roof of a house can be calculated with regular mathematics. The area of a complicated, non-spherical shape like the dome of the Houston Astrodome can be calculated by calculus. Architects designing such a building need to know the dome's area to determine the cost of materials and to figure the weight of the dome (with and without snow on it). The weight, of course, is needed for planning the strength of the supporting structure. Check out Figure 5.



Regular Mathematics Problem:
What's the roof area?

Calculus Problem:
What's the dome's area?

Figure 5: Real world examples showing the use of calculus

In irrigation channels the engineers have to design various kinds of shapes. The following figure shows one of them.

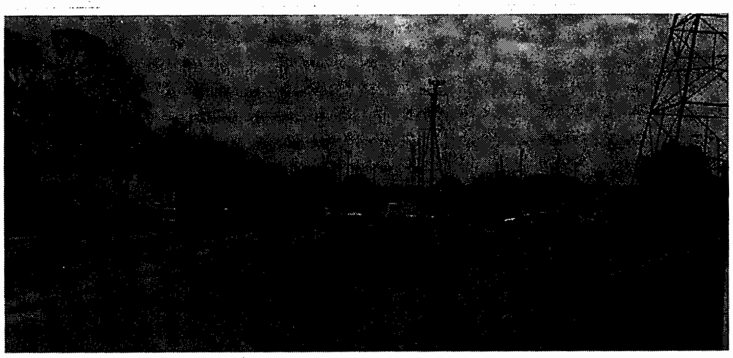
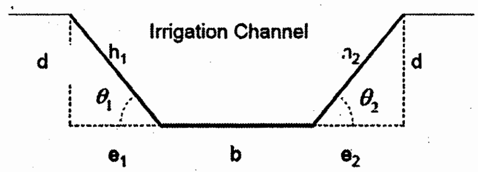


Figure 6: A water transport channel in an irrigation project



θ is the angle of inclination of each side. The other relevant dimensions are labeled on the figure.

Figure 7: A cross view structure of the channel shown in Figure 6

The cross view structure of the irrigation channel shown in Figure 6 becomes a trapezoidal channel of uniform depth d (Figure 7). To maintain a certain volume of flow in the channel, its cross- sectional area A is taken as a constant. In order to minimize the amount of concrete that must be used to construct the lining of the channel it should be used derivatively.

References

1). Theory and Problems of Calculus by Frank Ayres and Elliott Mendelson
2). Calculus I by Paul Dawkins
3). Calculus and Analytic Geometry by George B. Thomas, Jr. Ross L. Finney.
4). Application of Calculus by Dr. Manoj Chopra, P.E.
5). http://www.google.lk/#hl=en&source=hp&q

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