

An Enumerative Constraint Planning Framework for Airline Engineering Manpower Cost Optimization

WTS Rodrigo and WDT Fernando[#]

Department of Aeronautical Engineering, General Sir John Kotelawala Defence University, Sri Lanka

[#]dakshinafdo@kdu.ac.lk

Abstract: This paper discusses the formulation of a general enumerative constraint planning model relevant to different aspects of airline engineering workforce planning dimensions observed in the contemporary flight line operations. However, it is to be understood this is a general theoretical framework that needs to be custom-tailored to suit the diversified operational scenarios discussed therein.

Initially, evidential coherence of pre studied literature was comprehensively analyzed in ascertaining, understanding of predefined methodologies, and impediments of which. Whereupon the primary data elements are classified enlightened by said findings, and subsequently, relationships and the constraints related to data elements are discussed. It is followed by a discussion of the expected objectives and an employee-centered general model with the objective of direct or indirect cost optimization is formulated. Commuting on the contrived model, the amalgamation of a wide range of constraints has superseded the circumspection requirements of predefined models identified through literature. The general model signifies the complete aviation workforce planning problems from mediocre to prodigious cases.

Keywords: Enumerative constraint programming, Workforce Scheduling, Aviation management

Introduction:

Generally, workforce planning sequentially solve the interdependent decisions of staffing (i.e. estimating workforce capacity to meet the demand spanning through a longer horizon) and rostering (i.e. assigning employees to

planned shift schedules) (Van Den Bergh et al., 2013). These two phases are constrained by an array of constraints related to cost, fairness, regulations, and general HR policies. Due to the computational complexity of the capacity assignment problem, different methods are used in literature where heuristics and enumerative programming are the most prominent (Al-Thani et al., 2016; Jamili, 2017). The complex calculation of the optimal equilibrium between skill-specific workload and capacity assignment in terms of ECP is discussed in Cuevas et al., (2016). A mixed-integer program is used to assign multiskilled employees for short term demand fulfillment with a modified version of the general tour scheduling problem. This enables management to assign simultaneous shifts and days-off for the heterogeneous workforce to cater to the firm's workload demand.

Research Objectives

The objective of any workforce scheduling problem is to employ available scarce resources most efficiently while attaining the job in the best possible quality at an optimal cost. However, the secondary level planning objectives are employee satisfaction, fairness in job distribution, optimal productivity, etc.

- i. Minimization of operational cost without compensating operational reliability and quality.
- ii. Ensuring fairness in workload distribution (including unpopular shift distribution) among employees.

Literature Review

A diverse array of computational methods have been used to solve different workforce planning problems where enumerative methods and heuristics are extensively used due to problem complexity and the exponential scale of the planning problem. Kasirzadeh, Saddoune, & Soumis (2014) present a detailed review of crew scheduling models and methods discussed since the year 2014. Bechtold (1981) distinguish linear programming and construction-based heuristics to be the main solution methods. Nevertheless, integer programming, decomposition, linear programming, metaheuristics, manual solutions, implicit modeling, goal programming, working set generation and construction/improvement methodologies have also been used significantly (Deng and Lin 2011; Iide, Ryan, and Ehrgott 2010). Also, Lavoie et al. (1988) formulate a large-scale set covering problem with many columns where each represents a valid crew pairing, and the authors propose a continuous relaxation to solve a scenario inclusive of 329 segments of flight legs through column generation method based on generalized linear programming. Ryan (1992) examines a generalized set partitioning model for aircrew scheduling involving more than 650 constraints and 200,000 binary variables. Yan & Chang (2002) discuss cockpit crew planning in specific as the salary and remuneration of the pilots cover a significant portion of overall crew costs where a set partitioning model is formulated to solve it through column generation. Deng & Lin (2011) use an ant-colony optimization-based algorithm to solve airline crew scheduling problems with various enumerations. Mercier & Soumis (2007) solve the optimal crew-scheduling problem along with aircraft routing and retiming. The above references confirm the fact that a vast majority of capacity planning and crew scheduling research are solved through complex mathematical methodologies. As mentioned above, staffing phase estimates capacity according to the demand which

generally spans through a longer planning horizon. The rostering phase assigns employees to planned shifts on a weekly or monthly basis (Van Den Bergh et al., 2013). The limiting constraints of these two processes are different as the scheduling constraints are process-oriented and staffing constraints depend on employee preferences and HR policies. Generally, mixed-integer linear programming (MILP), metaheuristics are used to solve these problems separately or simultaneously (Pinedo et al., 2015). Most MILP software use branch and bound enumeration framework to solve problems through LP relaxations. Also, integer programming column generation through a branch and price enumeration framework has emerged as a promising method for mid-size scheduling problems. However, real-world WP problems can hardly be solved optimally due to the NP-hardness of the problems (Heimerl & Kolisch, 2010). Therefore in recent literature, metaheuristic approaches also have gained popularity in finding feasible solutions within a reasonable computation time. It is also to be noted that metaheuristics, unlike the enumerative constraint programming, does not ensure a quality solution due to its implicit neighborhood search patterns (Brucker et al., 2011).

Methodology:

Initially, primary data elements are classified, and subsequently, relationships and the constraints related to data elements are discussed. It is followed by a discussion of the expected objectives and an employee-centered general model with the objective of direct or indirect cost optimization is formulated. Dependent and independent are the main two types of data elements in any WP problem, and a proper understanding of the distinction is essential in reducing the problem size, even though it seems trivial. At times, the data elements which are seemingly independent are dependent intrinsically (Brucker et al., 2011). On the other hand, some seemingly dependent elements have nothing to do with the other

elements. For a large airline sometimes it would be possible to divide the operational demand, terminal wise, to obtain an independent and smaller planning problem. As these solutions are mainly computer-based, the downsizing may not help the modeling problem but reduce computational time and enhance solution quality (Defraeye & Van Nieuwenhuyse, 2016; Van Den Bergh et al., 2013). The problem complexity dilution is always preferred than reducing the problem space especially for large scale planning problems (Wiegmann et al., 2017). The essential data elements of a general capacity planning problem are as below;

Data elements

1) *Item(s)* - The objects which are to be scheduled are items (e). It is important to note that at a time an “item” can only be in one place (Brucker et al., 2011). It has a unique identification supported by additional information. In most of the cases, an “item” in WP is an employee with different skills.

2) *Block of Time (BOT)* - A BOT(t) is a period during which an “item’s” operation is planned. It must have a determined duration (i.e., start and end) and has properties like cost where a BOT is more generic than a “shift” as it may represent traveling time, meal brakes, etc (RAO, 1975).

3) *Task/Job* - A task/Job represents the composition of an “items” throughout a defined duration (BOT). An example of the simplest type of composition is the “requirement of two technicians for the night shift from 0000hrs to 0800hrs”. However when logical operations like “and, or, not” are incorporated, the compositions get more complicated (Ernst et al., 2004). When the number and the type of items increase composed of different BOTs, the problem is no more trivial.

4) *Costs* - Every assignment of an item to a BOT will incur a cost. The costs associated with different pairings of items and BOTs vary (Periyar Selvam et al. 2013; Saltoğlu, Humaira, and Inalhan 2016). For instance, the cost

associated with assigning employees for a day shift is not equivalent to night or weekend shift assignment as they are relatively unpopular and therefore costlier. The cost of assigning (e) item to a (t) BOT is denoted by (c_{et}).

5) *Decision variables* - Assigning an optimal number of items to different BOTs at a minimal cost is the solution of a workforce plan. Such a solution can be represented by a binary variable set as below;

$$x_{et} = \begin{cases} 1 & \text{if (e) assigned to (t)} \\ 0 & \text{if otherwise} \end{cases} \quad \text{where } x_{et} \in X$$

and X denotes all decision variables

Constraints

In workforce planning, a diverse array of constraints are to be dealt with, and the same are categorized as “hard” and “soft” constraints followed by a set of variants.

1) *Hard constraints* - The ones which cannot be violated are the hard constraints. For instance, one employee can work only in one aircraft at any given time. This constraint for item (e) can be formulated as below;

$$\sum_{t \in T} x_{et} \leq 1 \quad \text{Where for } \forall T \quad (1)$$

If (T) is a set of BOTs which overlap with each other, according to the inequality only one BOT can be selected at most as 0 & 1 are the only possible values for vthe ariable (x_{et}).

2) *Soft constraints* - Unlike hard constraints, the solutions are acceptable even with violated soft constraints. Nevertheless, it affects the solution quality and problem objectives. Hence it is vital to evaluate the level of violation and to understand how far a soft constraint can be bent in attaining a feasible solution. This could be achieved by associating a “violation coefficient.” (v_c) with each soft constraint(c). It is to be further noted that ($v_c = 0$) when the soft constraint(c) is perfectly met and (th_c) is the threshold that defines an unacceptable level of violation.

c -Denotes soft constraint

v_c -Denotes violation coefficient associated to (c)

$v_c(X)$ -Denotes violation incurred to the solution (X) by soft constraint (c)

th_c -The denoted threshold of the maximum tolerated level of(c)'s violation

The above targets to get a feasible solution for(X) where $v_c(X) < th_c$ for every (c) and preferably, smaller the value of $v_c(X)$ is better. When there are more than one soft constraints a measure to be identified (C_{sof}) to represent the sum of violations. For optimal results C_{sof} should be minimal.

3) *Sequence constraints* - For a scheduling problem to be practical, one has to restrict BOTs with sequence constraints. When an employee is considered, it is evident that another 12-hour shift cannot immediately follow one shift with a duration of 12 hours. It is both against regulations and human endurance levels especially during unfavorable shifts (i.e. Nights & Weekends). For each BOT (t) one could define a set (T_t) which represents BOTs that could follow(t). If (T_t) represents all BOTs except BOT(t_1), then(t) could not be followed by (t_1) in the scheduling sequence for the same item (i.e., employee). Therefore a different sequence needs to be represented by a set (T_t) with only one BOT (t_2) where the workforce planner has no choice but to schedule (t) followed by(t_2). (T_t) Represents a set of BOTs that could be scheduled after BOT(t). Hence the formulation of the sequence constraint is as follows;

$$\sum_{t' \in T_t} x_{et'} \geq x_{et} \text{ for } \forall e, t \quad (2)$$

Generally, the sequence constraints are hard, and the above inequality signifies that if item (e) is assigned to BOT (t) (then(x_{et}) = 1), at least a single BOT in T_t should be assigned to (t). So it is evident that the sum is always nonnegative.

4) *Counting constraints* - These are highly flexible type of constraints which count different things over variable BOTs. In most occasions, the counting results must fall within an acceptable range for the solution to be optimal. For example in most service

organizations, accepted range of working hours span from 40 – 50 hours where the planning horizon would be seven days or a standard week starting from Monday. In addition, some other examples for counting constraints are specified ranges of unpopular shifts assigned to an employee over a planning horizon of one month and the available paid holidays for one year. Majority of the counting constraints are soft constraints and can be denoted by the function (f_c).

5) *Work constraints* - The primary purpose of WP is to get the work done in the best possible way. Therefore, work constraints are critical and may be either hard or soft depending on the type of problem. Mainly the work constraints are exclusive to individual problem settings and vary according to the definition of the job requirement. For example (e^{\min}) is the least amount of items required to fulfill a job and (e^{\max}) is the maximum items needed. (E) Is the set of all possible items which could be scheduled for the job and there is a set of BOTs (T) which are available to carry out the job and also it entirely cover job duration. In such setting the work constraint could be formulated as below;

$$e^{\min} \leq \sum_{t \in T, e \in E} x_{et} \leq e^{\max} \quad (3)$$

6) *Compatibility constraints* - Some items cannot be assigned together to a same BOT due to compatibility issues. Such constraints are considered as compatibility constraints. For instance, there might be two employees who does not get along well with one another. If (I) denotes a set of mutually incompatible items and (J) represents incompatible sets of item (I) then the compatibility constraint formulation is as follows;

$$\sum_{e \in I} x_{et} \leq 1 \text{ where } \forall I \in J, \forall t \quad (4)$$

7) *Internal and external constraints* - Internal constraints are demarcated by the nature of items which are to be scheduled while external constraints are governed by external environmental influences like administrative relations, labour laws, etc. For example the constraint that one employee could only work

in one place during a given BOT is an external constraint while a maximum number of work hours authorized by labour regulations are external constraints.

Objectives formulation

The objective function is the main element of a WP problem. This may comprise of multiple parts depending on the size and scope of the problem. The following can be highlighted as the main elements of a generalized objective function.

1) *Cost* - Minimization of operational cost without compensating reliability and quality is the primary objective of the majority of workforce planning problems. Here it is to be noted that cost and capacity are interrelated in the optimal capacity planning problem. The cost (C) is the sum of the set of all (X) assignments and is denoted as below;

$$C = \sum_{e,t} x_{et} C_{et} \quad (5)$$

2) *Fairness* - Fairness in workload and unpopular shift distribution among employees is important. When doing this one needs a measure to evaluate the level of the unpopularity of a BOT (i.e., a shift) or the number of jobs assigned to a BOT. If (u_t) denote the measure of the unpopularity of a given BOT (t) we understand when (u_t) increase the unpopularity of the BOT (t) will increase accordingly. Therefore, when considering one employee, the unpopularity of a BOT (i.e. a shift) is formulated as below;

$$U_e = \beta_e \sum_t x_{et} u_t \quad (6)$$

Here (β_e) is a workload coefficient as the unpopular shift distribution has to be inversely proportionate to the individual workload of employees. When considering all employees in an organization, if all values of (U_e) are equal then a 100% fair job distribution prevails. However, this is far from reality about any service organization. So an overall fairness measure (D_f) needs to be defined, and the difference between the best case and the worst case should be minimized.

$$D_f = \sum_t u_{t(\max)} - u_{t(\min)} \quad (7)$$

If the job distribution is 100% fair the (D_f) will be zero and lower the value of (D_f) it is preferred as a better solution. Instead of the above, the standards deviation of (D_f) can also be considered.

3) *Violation of soft constraints* - The term “soft constraints” itself, indicate that satisfying all soft constraints are impossible. It is not practical to add them as constraints. Therefore a measure is to be defined how these soft constraints are respected while trying to optimize them. As a result, the soft constraints set becomes a sub-portion of the objective function. If the cost of all soft violations are denoted by (C_s) ;

$$C_s = \sum_c f_c (X) \quad (8)$$

The above represents the summation of all soft constraints

Mathematical model formulation

As highlighted above, there are three elements to minimize; the cost associated on assignments of items to BOTs denoted by (C) , measurement of overall job distribution unfairness (D_f) and measurement of cumulative soft violations (C_s) . When considering an ideal situation, one could express measures of unfairness (D_f) and soft violations (C_s) in same units of assignment cost (C) by multiplying them by two constants (α_f) for fairness and (α_s) for soft violations. Then the overall objective function will appear as below;

$$F(X) = C(X) + \alpha_f D_f(X) + \alpha_s C_s(X) \quad (9)$$

The following general model is a combination of all the elements discussed above. The model addresses the noncyclic homogeneous WP problems and can be adapted to solve a variety of real-world workforce planning issues. The formulation is as follows;

$$\text{Minimize } F(X) = C(X) + \alpha_f D_f(X) + \alpha_s C_s(X) \quad (10)$$

$$\text{s.t. } \sum_{t \in T} x_{et} \leq 1 \quad \forall T, \forall e \quad (11)$$

$$v_c(X) < th_c \quad \forall c \quad (12)$$

$$\sum_{t \in T_t} x_{et} \geq x_{et} \text{ for } \forall e, \forall t \quad (13)$$

$$\sum_{e \in I} x_{et} \leq 1 \text{ where } \forall I \in J, \forall t \quad (14)$$

$$e^{\min} \leq \sum_{t \in T, e \in E} x_{et} \leq e^{\max} \quad (15)$$

$$x_{et} \in \{0,1\} \quad (16)$$

Discussion:

Equation (10) is the objective function where it minimizes the overall cost associated item assignments, fairness violations, and soft violations. It is to be noted that Equation (11) denotes (T) set of BOTs which overlap with each other, and according to the inequality only one BOT can be selected at most as 0 & 1 are the only possible values for the variable(x_{et}). Equation (12) denotes a feasible solution for(X) where $v_c(X) < th_c$ for every (c) and preferably, smaller the value of $v_c(X)$ is better. Equation (13) denotes that if item (e) is assigned to BOT (t) (then(x_{et}) = 1), at least a single BOT in T_t should be assigned to (t). so it is obvious that the sum is always nonnegative. Equation (14) denotes (I) a set of mutually incompatible items and (J) represents incompatible sets of the item (I) then the compatibility constraint formulation is given in the equation. Equation (15) denotes the work constraints. For an example (e^{\min}) is the least amount of items required to fulfill a job and (e^{\max}) is the maximum items needed. (E) Is the set of all possible items which could be scheduled for the job and there is a set of BOTs (T) that are available to carry out the job and also it entirely covers job duration. Equation (16) denotes,

$$x_{et} = \begin{cases} 1 & \text{if (e) assigned to (t)} \\ 0 & \text{if otherwise} \end{cases} \quad \text{where } x_{et} \in X$$

X and X denotes all decision variables

where $x_{et} \in X$ and X denotes all decision variables.

References:

Al-Thani, N. A., Ben Ahmed, M., & Haouari, M. (2016). A model and optimization-based heuristic for the operational aircraft maintenance routing problem. *Transportation*

Research Part C: Emerging Technologies, 72(November)

Bechtold, S. E. (1981). Work force scheduling for arbitrary cyclic demands. *Journal of Operations Management*, 1(4), 205–214.

Beliën, J., Demeulemeester, E., De Bruecker, P., Van Den Bergh, J., & Cardoen, B. (2013). Integrated staffing and scheduling for an aircraft line maintenance problem. *Computers and Operations Research*, 40(4), 1023–1033.

Brucker, P., Qu, R., & Burke, E. (2011). Personnel scheduling: Models and complexity. *European Journal of Operational Research*, 210(3), 467–473.

Cuevas, R., Ferrer, J. C., Klapp, M., & Muñoz, J. C. (2016). A mixed integer programming approach to multi-skilled workforce scheduling. *Journal of Scheduling*, 19(1), 91–106.

Defraeye, M., & Van Nieuwenhuysse, I. (2016). Staffing and scheduling under nonstationary demand for service: A literature review. *Omega (United Kingdom)*, 58, 4–25.

Deng, G.-F., & Lin, W.-T. (2011). Ant Colony Optimization-based Algorithm for Airline Crew Scheduling Problem. *Expert Syst. Appl.*, 38(5), 5787–5793.

Dijkstra, M. C., Kroon, L. G., van Nunen, J. A. E. E., & Salomon, M. (1991). A DSS for capacity planning of aircraft maintenance personnel. *International Journal of Production Economics*, 23(1–3), 69–78.

Ernst, A. T., Jiang, H., Krishnamoorthy, M., Owens, B., & Sier, D. (2004). An Annotated Bibliography of Personnel Scheduling and Rostering. *Annals of Operations Research*, 127(1–4), 21–144.

Heimerl, C., & Kolisch, R. (2010). Scheduling and staffing multiple projects with a multi-skilled workforce. *OR Spectrum*, 32(2), 343–368.

Jamili, A. (2017). A robust mathematical model and heuristic algorithms for integrated aircraft routing and scheduling, with consideration of

- fleet assignment problem. *Journal of Air Transport Management*, 58, 21–30.
- Kasirzadeh, A., Saddoune, M., & Soumis, F. (2017). Airline crew scheduling: models, algorithms, and data sets. *EURO Journal on Transportation and Logistics*, 6(2), 111–137.
- Kilincli Taskiran, G., & Zhang, X. (2017). Mathematical models and solution approach for cross-training staff scheduling at call centers. *Computers and Operations Research*, 87, 258–269.
- Lavoie, S., Minoux, M., & Odier, E. (1988). A new approach for crew pairing problems by column generation with an application to air transportation. *European Journal of Operational Research*, 35(1), 45–58.
- Maenhout, B., & Vanhoucke, M. (2018). A perturbation matheuristic for the integrated personnel shift and task re-scheduling problem. *European Journal of Operational Research*, 269(3), 806–823.
- Mercier, A., & Soumis, F. (2007). An integrated aircraft routing, crew scheduling and flight retiming model. In *Computers and Operations Research* (Vol. 34, Issue 8, pp. 2251–2265).
- Periyarselvam, U., Tamilselvan, T., Thilakan, S., & Shanmugaraja, M. (2013). Analysis on Costs for Aircraft Maintenance. *Research India Publications*, 3(3), 177–182.
- Pinedo, M., Zacharias, C., & Zhu, N. (2015). Scheduling in the service industries: An overview. *Journal of Systems Science and Systems Engineering*, 24(1), 0–48.
- RAO, G. L. Lilien. A. A. G. (1975). A model for manpower management. *Management Science*, 21(12), 1447–1457. <https://doi.org/10.1287/mnsc.21.12.1447>
- Ryan, D. M. (1992). The Solution of Massive Generalized Set Partitioning Problems in Aircrew Rostering. *Journal of the Operational Research Society*, 43(5), 459–467.
- Saltoğlu, R., Humaira, N., & Inalhan, G. (2016). Aircraft Scheduled Airframe Maintenance and Downtime Integrated Cost Model. In *Advances in Operations Research* (Vol. 2016).
- Van den Bergh, J., Belien, J., De Boeck, L., & Demeulemeester, E. (2013). A rolling horizon framework for an aircraft line maintenance problem. *EURO-INFORMS - Rome 2013*, 11.
- Van Den Bergh, J., Beliën, J., De Bruecker, P., Demeulemeester, E., & De Boeck, L. (2013). Personnel scheduling: A literature review. *European Journal of Operational Research*, 226(3), 367–385.
- Weide, O., Ryan, D., & Ehrgott, M. (2010). An iterative approach to robust and integrated aircraft routing and crew scheduling. *Computers and Operations Research*, 37(5), 833–844. <https://doi.org/10.1016/j.cor.2009.03.024>
- Wiegmann, P. M., de Vries, H. J., & Blind, K. (2017). Multi-mode standardisation: A critical review and a research agenda. *Research Policy*, 46(8), 1370–1386.