Modelling and Forecasting Monthly Petroleum Crude Oil Prices Using a Hybrid Time Series Model

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Abstract— *Crude oil is a naturally occurring resource composed of hydrocarbons and other organic material. Crude oil price exert a great impact on the global economy. Therefore, modelling and forecasting crude oil prices are essential tasks for government policy makers, investors and even researchers. The objective of this study is to develop a more accurate time series model for the monthly crude oil prices. The data consisted of 241 monthly observations of crude oil prices spanning from April, 1999 to April, 2019. Since the time series of monthly crude oil prices was non-stationary, the first difference data set was used where it proved the stationary by both graphical and theoretical techniques. The best Autoregressive Integrated Moving Average (ARIMA) model was selected by using the criteria of Akaike Information Criterion (AIC), Schwarz Information Criterion and Hannan-Quinn Information Criterion after testing for different ARIMA models. Since Auto Regressive Conditional Heteroscedasticity (ARCH) effect was presented in the crude oil price time series, a suitable model was fitted to capture the volatility clustering. The best model was identified by the lowest AIC values after testing for various ARCH and GARCH (Generalized ARCH) models. Hence ARIMA (1, 1, 0) + GARCH (1, 1) was found to be the best model with lesser root mean squared error of 4.3017. It can be concluded that the combination of ARIMA and GARCH models in handling volatility made hybrid models as the most suitable for analysis and forecasting crude oil prices.*

Keywords— crude oil price, GARCH, ARIMA

I. INTRODUCTION

Crude oil, one of the most important energy resource in the world, exhibits wide fluctuation. Its fluctuation has significant effects on the sales and profits of major industries worldwide, and influence capital budgeting plans as well as the economic instability in both oil exporting and oil consuming countries. So far, it remains the world's leading fuel, with nearly one-third of global energy consumption.Therefore, modelling and forecasting oil prices are important to economic agents and policy makers. (Pavlova, et al., 2017)

The world's environment is affected by the oil price falling. With the drop of oil prices, the fuel bills are lowered. As a result, consumers are very likely to use more oil and thus increase the carbon emission. In addition, there is less incentive to develop renewable and clean energy resources. On the other hand, sustained low oil prices could lead to a drop in global oil and gas exploration and exploitation activities (Baumeister & Kilian, 2014).

There is no doubt that crude oil price forecasts are very useful to industries, governments as well as individuals. Thus, forecasting crude oil prices has been the subject of research by both academia and industry. Many methods and approaches have been developed for predicting oil prices. However, due to the high volatility of oil prices (Regnier, 2007), it remains one of the most challenging forecasting problems.

Crude oil price dynamics and evolution can be studied using a stochastic modelling approach that captures the time dependent structure embedded in the time series crude oil price data. The Autoregressive Integrated Moving Average (ARIMA) popularly known as Box-Jenkins Methodology (Ljung & Box, 1978) and the autoregressive conditional heteroscedasticity (ARCH) models, with its extension to generalized autoregressive conditional heteroscedasticity (GARCH) models as introduced by Engle (1981) and Bollerslev (1986) respectively accommodates the dynamics of conditional heteroscedasticity (the changing variance nature of the data). Heteroscedasticity affects the accuracy of forecast confidence limits and thus has to be handled properly by constructing appropriate non-constant variance models (Amos, 2010).

In real life, financial data variance changes with time (a phenomenon defined as heteroscedasticity), hence there is a need of studying models which accommodates this possible variation in variance. In considering the issue crude oil price modelling and forecasting, this work consequently intends to also use the Box-Jenkins methodology (ARIMA) and autoregressive conditional heteroscedasticity (ARCH) models with its extension to generalized ARCH (GARCH) models to model and accommodate the dynamics of conditional heteroscedasticity in crude oil price data. Therefore, the objectives of this study include developing a time series model for the crude oil price data, determining the accuracy of the fitted model and determining future crude oil prices.

II. METHODOLOGY

A. *Data*

Monthly observations of crude oil price spanning are used for the study, where data during April, 1999 to March, 2017 are used to train the model and data during April, 2017 to April, 2019 are used to test the model. The data have been collected from the World Bank data catalogue (Anon., 2019).

B. Theory

1) Stationary Time series: A time series is strictly stationary if the properties of the time series do not change when the time origin is changed. That is, when the joint probability distribution of the observations y_t, y_{t+1}, y_{t+2} ..., $y_{t+n}$is the same as the joint probability distribution of y_{t+k}, y_{t+k+1}, y_{t+k+2}, ...,y_{t+k+n}. A time series is said to be weakly stationary (second order stationary) if it has a constant mean and if auto covariance function depends only on the lag. Augmented Dickey-Fuller (ADF) test, Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test and PhillipsPerron (PP) test have been used to check the stationary of the models.

2) Autoregressive integrated moving Average (ARIMA) process: A time series should be made stationary before fitting an ARMA model. In order to make a time series stationary differencing technique can be used where the trend gets eliminated. So that the first differenced time series or a higher-order differenced time series is stationary. Then we call that process as an ARIMA (p,d,q) process where d denotes the d^{th}differenced stationary time series.

ARIMA (p,d,q) can be written as,

$$\emptyset(B)(1-B)^d X_t = \delta + \theta(B)Z_t \quad (1)$$

Where Φ(B) the polynomial of AR is process and Θ(B) is the polynomial of MA process. And B is the backshift operator.

3) ARCH/GRACH Model: The Autoregressive Conditional Heteroscedasticity (ARCH) model concentrates on the volatility dynamic.

The general form of the ARCH model is:

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \cdots + \alpha_q\epsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 \quad (2)$$

Where $\alpha_0$is mean, α_i is conditional volatility and ε_{t-i} is white noise representing residuals of time series.

The modification of ARCH model was introduced as Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model; that synchronized both lagged squared residuals and lagged variances. In this way GARCH model is allowed to be dependent on both recent variances of itself side by side with past shocks, so at the end it will provide us with volatility clustering.

In general, the GARCH (p, q) model is presented in the following formula:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j\, \varepsilon_{t-j}^2 + \sum_{i=1}^{p} \beta_j\, \sigma_{t-j}^2 \quad (3)$$

Where i=0,1,2,3,… p, conditional volatility, ω ,α_j and β_j are non-negative constants with α_j+ β_j<1 it should be near to unity for an accurate model, ε_{t-j} is residuals and it is lagged conditional volatility. And the last part of the formula is the main difference in applying both ARCH and GARCH models. Hence, α_j and ε_{t-j}^2 are ARCH components and βj and σ_{t-j}^2 are GARCH components. In addition, both ARCH and GARCH models depend on a major assumption that is; all of the shock effects on volatility have a symmetric distribution.

4) Augmented Dickey-Fuller (ADF) Test: Augmented Dickey-Fuller test can be used to identify the existence of unit roots in the series.

$$y_t = c + \delta_{1t} + \emptyset y_{t-1} + \beta_1\delta y_{t_1} + \cdots + \beta_p\delta y_{t_p} + \epsilon_t \quad (4)$$

Where, δ is the differencing operator, such that, $\delta y_t = y_t - y_{t-1}$

H_0: ϕ=1

H_1 : ϕ<1

This hypothesis can also be interpreted as H_0 is the series has a unit root and H_1 is series is stationary.

*5) Kwiatkowski – Phillips-Schmidt –Shin (KPSS) Test:*This test identifies whether a time series is trend stationary or whether the series is a non-stationary unit root process.

$y_t = c_t + \delta_t + u_{1t} c_t = c_{t-1} + u_{2t}$
(5)

Where, δ is the trend coefficient, u_{1t} is a stationary process and u_{2t} is an independent and identically distributed process with mean 0 and variance σ^2.

$H_0: \sigma^2 = 1$
$H_1: \sigma^2 < 1$

*6) Phillips-Perron (PP) Test:*Phillips-Perron test assess the null hypothesis of a unit root in a univariate time series *y*. All tests use the model:

$$y_t = c + \delta_t + a y_{t-1} + e(t) \quad (6)$$

The null hypothesis restricts a = 1. Variants of the test, appropriate for series with different growth characteristics, restrict the drift and deterministic trend coefficients, *c* and *δ*, respectively, to be 0. The tests use modified Dickey-Fuller statistics to account for serial correlations in the innovations process *e*(*t*).

7) Mean Squared Error: Mean Squared error calculates the average of squared errors over the sample period.
The equation used to calculate mean square error is as follows:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(a_t - f_t)^2 (7)$$

Where n is the sample size, a_t is the actual value and f_t is the forecasted value.

III. RESULTS

A. Descriptive Analysis

Figure 1 shows the monthly OPEC reference basket crude oil price from April, 1999 to April, 2019.

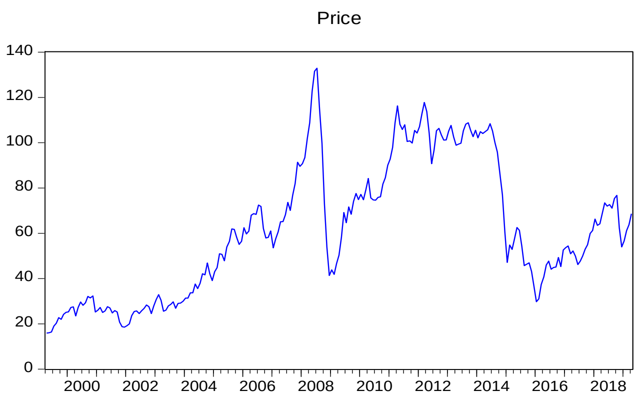


Figure 1: Time plot of the monthly crude oil price

The above time series plot exhibits the phenomenon of volatility clustering that shows wide swings for an extended time period followed by periods in which there is relative calm. Over the period of study, the crude oil price has been increasing, that is, showing an upward trend, in a fluctuation pattern. By observing the behaviour of the price we can say that the dataset is noisy and nonstationary. Formal tests will be done in the latter parts of this study to identify the non-stationarity of the time series.

B. Stationary of the time series

By observing Figure 1 we identified that the oil price series is non-stationary. In order to verify the claim, more sophisticated tests such as unit root tests have been carried out.

According to the ADF test, the p value is 0.1361 which is greater than 0.05 significance level. Hence we do not reject the null hypothesis that crude oil price has a unit root at 5% level of significance. Hence we accept that there is a unit root. Since there exists a unit root, the series is non-stationary.

After testing using KPSS test, the test statistic value is 0.9489 and it is greater than the critical value of 5% level. Hence we reject the null hypothesis that the crude oil price series is stationary. Hence from the above test we can conclude that the price series is not stationary at 5% level of significance.
According to the PP test,the p value is 0.2121 which is greater than 0.05 significance level. Hence we do not reject the null hypothesis that crude oil price has a unit root at 5% level of significance. Hence we accept that there is a unit root. Since there exists a unit root, the series is non-stationary.

The non-stationary of the time series was identified by graphical inspection. Then the non-stationary of the series was proved by ADF, KPSS and PP tests. Hence it was concluded that the OPEC reference basket crude oil price series is non-stationary.

Since it is necessary to have a stationary time series to fit a model, a data transformation is carried out to identify a stationary time series of the crude oil. First difference time series was considered for the model fitting and the stationarity of the first difference time series was also tested.

C. First Difference Time Series

Figure 2 illustrates the first difference time series of crude oil price.

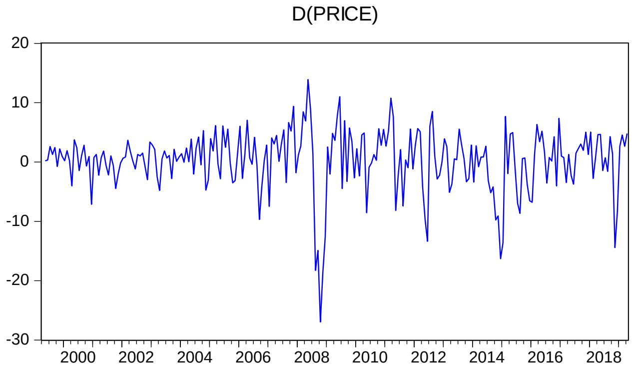


Figure 2: Plot of first difference of crude oil price

By observing the above Figure 2, we can say that the dataset is having a constant variance and not having an upward or downward trend. That is, the first differenced dataset is stationary. ADF, KPSS and PP tests were carried out to identify the stationarity of the first differenced time series.

The stationary of the first difference time series was identified by graphical inspection. Then the stationarity of the series was proved by all the three tests namely ADF,

KPSS and PP tests. The test results are shown in the Table 1.

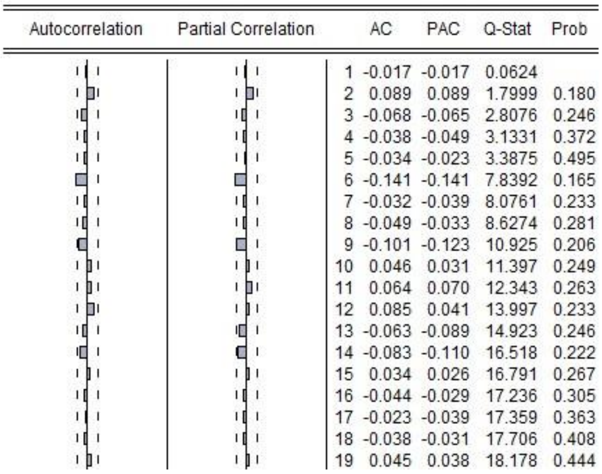
Table 1: Unit root test results for first difference crude oil prices

Unit Root Test	Test Statistic	Result
ADF Test	0.0000	Stationary
KPSS Test	0.0812	Stationary
PP Test	0.0000	Stationary

Hence the first differenced time series was taken for the model fitting.

D. Model Identification and Selection

According to the ACF and PACF plots in Figure 3, different ARIMA models were tested.



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.017	-0.017	0.0624	
		2	0.089	0.089	1.7999	0.180
		3	-0.068	-0.065	2.8076	0.246
		4	-0.038	-0.049	3.1331	0.372
		5	-0.034	-0.023	3.3875	0.495
		6	-0.141	-0.141	7.8392	0.165
		7	-0.032	-0.039	8.0761	0.233
		8	-0.049	-0.033	8.6274	0.281
		9	-0.101	-0.123	10.925	0.206
		10	0.046	0.031	11.397	0.249
		11	0.064	0.070	12.343	0.263
		12	0.085	0.041	13.997	0.233
		13	-0.063	-0.089	14.923	0.246
		14	-0.083	-0.110	16.518	0.222
		15	0.034	0.026	16.791	0.267
		16	-0.044	-0.029	17.236	0.305
		17	-0.023	-0.039	17.359	0.363
		18	-0.038	-0.031	17.706	0.408
		19	0.045	0.038	18.178	0.444

Figure 3: ACF and PACF of first difference crude oil price

Table 2 shows the AIC, SIC and HQC test values for some of the fitted ARIMA models.

Table 2: Results of ARIMA model identification and selection

Model	AIC	SIC	HQC
ARIMA (1,1,0)	**6.0079**	**6.0549**	**6.0269**
ARIMA (1,1,1)	6.0159	6.0789	6.0413
ARIMA (1,1,2)	6.0082	6.0709	6.0335
ARIMA (1,1,3)	6.0104	6.0731	6.0357
ARIMA (1,1,4)	6.0155	6.0782	6.0409
ARIMA (2,1,0)	6.1573	6.2044	6.1763
ARIMA (2,1,1)	6.011	6.0737	6.0363
ARIMA (2,1,2)	6.1609	6.2237	6.1863
ARIMA (2,1,3)	6.16	6.2227	6.1853
ARIMA (2,1,4)	6.1595	6.2222	6.1848
ARIMA (3,1,0)	6.2049	6.252	6.2239
ARIMA (3,1,1)	6.0611	6.1238	6.0864
ARIMA (3,1,2)	6.1581	6.2208	6.1834
ARIMA (3,1,3)	6.2108	6.2735	6.2361
ARIMA (3,1,4)	6.2081	6.2708	6.2334

Table 2 tested fifteen models with low AIC, HQC and SIC which is common in ARIMA modelling and find the best models among them. ARIMA (1, 1, 0) model was selected as the best model among fitted models because it has minimum AIC, HQC and SIC.

Table 3: Result of ARIMA (1, 1, 0) model estimation

Parameter	Coefficient	Std. Error	Zstatistic	P-value
δ	0.1508	0.6242	0.2416	0.8093
AR (1)	0.4223	0.0418	10.1144	0.0000

The estimates of the parameters of the model, shown in Table 3, indicates that AR (1) model is significant at the 0.05 significance level.

The equation for ARIMA (1, 1, 0) model is as follows.

$x_t - x_{t-1} = 0.1508 + 0.4223\,(x_{t-1} - x_{t-2})$(8)

E. Performance Evaluation of ARIMA (1, 1, 0) model

The best model is next tested for adequacy using different diagnostic tests.

The autocorrelations and partial autocorrelations are not zero at all lags and the *Q*-statistics are significant in the given Figure 3. Hence we can conclude that serial correlations are present at the residuals.

According to the normality test, the probability value is greater than 0.05 significance level. Hence we do not reject the null hypothesis that residuals are normally distributed 5% level of significance. Therefore, the errors are normally distributed.

Heteroscedasticity test was performed in the dataset to identify whether there is any ARCH effect present in the dataset (Eagle, 1982). The result is shown in Table 4.

Table 4: Heteroscedasticity test of ARIMA (1, 1, 0) model

Test	Test Statistic	P-value
ARCH	4.603643	0.0330

The test shows that there is ARCH effect present. Hence the residuals have no constant variance.

F. Modelling for Volatility

After building the ARIMA model for estimating mean, the volatility has been modelled using both ARCH and GARCH models.

Table 5: Table of AIC values for different ARCH models

Model	AIC
ARCH (1)	5.8807
GARCH (1,0)	5.8497
GARCH (1,1)	**5.7514**
GARCH (2,0)	5.8594
GARCH (2,1)	5.7605
GARCH (2,2)	5.7699

Table 5 indicates the lowest AIC values for few different ARCH models after testing for various ARCH and GARCH models.

According to Table 5, the minimum AIC value is present at the GARCH (1,1) model. Therefore, the appropriate model is GARCH (1,1). Table 6 shows the parameter estimation for GARCH (1, 1) model.

Table 6: Parameter estimation for ARIMA (1, 1, 0) + GARCH (1, 1) model

Parameter	Coefficient	Std. Error	Zstatistic	P-value
δ	0.3391	0.3187	1.0641	0.2873
AR (1)	0.1895	0.0758	2.5011	0.0124
alpha (0)	0.9329	0.6144	1.5184	0.1289
alpha (1)	0.2561	0.0887	2.8867	0.0039
beta (1)	0.7274	0.0926	7.8555	0.0000

G. Performance Evaluation of GARCH (1, 1) model

In the pre-estimation analysis, the ARCH test indicated rejection of the null hypothesis showing significant evidence in support of ARCH effects. After applying the heteroscedasticity test for the hybrid model, following results were observed.

Table 7: Heteroscedasticity test of ARIMA (1, 1, 0) + GARCH (1, 1) model

Test	Test Statistic	P-value
ARCH	0.115316	0.7345

According to Table 7 it can be concluded that there are no any ARCH effects left (no heteroscedasticity) after testing for the hybrid model.

The equation for full model of ARIMA (1, 1, 0) + GARCH (1, 1) is as follows.

$$x_t - x_{t-1} = 0.3391 + 0.1895(x_{t-1} - x_{t-2}) + 0.9329 + 0.2561\varepsilon_{t-1}^2 + 0.7274\sigma_{t-1}^2$$

(8)

Therefore, we proceed to use the models to forecast future values of the Crude Oil Prices.

H. Forecasting the Time Series

Figure 4 shows the two years' samples forecast. It can be confirmed that the forecasted values are close to the actual values, thus, the model adequately fits the data well (Alquist, et al., 2011; Shuang&Yalin, 2017).

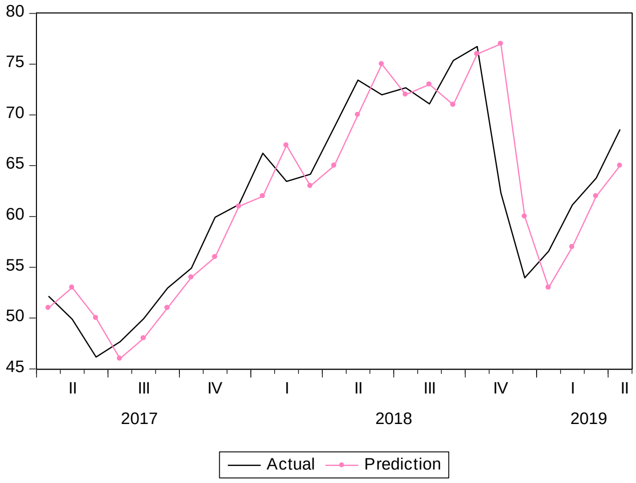


Figure 4: Plot of actual and predicted crude oil price from April, 2017 to April, 2019

For ARIMA (1, 1, 0) + GARCH (1, 1) model, Root Mean Squared Error, Mean Absolute Error and Mean Absolute Percentage Error values are shown in the following table.

Table 8: Summary measures of the forecasted model

Measure	Value
Root Mean Squared Error	4.301703
Mean Absolute Error	3.016927
Mean Absolute Percentage Error	4.958481

According to Table 8 it can be concluded that the model forecasted well.

According to the results, the appropriate model for estimating mean of the monthly OPEC reference basket crude oil price is ARIMA (1,1,0) and the appropriate model for modelling the volatility of the monthly OPEC reference basket crude oil price is GARCH (1,1).

IV. DISCUSSION AND CONCLUSION

The crude oil price data set was initially non stationary where volatility clustering can be seen after looking for the time plot of crude oil price. Then first difference of crude oil price data was tested for the stationary of the data set where it satisfies the stationary conditions.

Afterwards different combinations of ARIMA models were tested according to the cut off values of the ACF and PACF graphs. Among all the ARIMA models, ARIMA (1, 1, 0) was chosen as the best ARIMA model with minimum AIC, HQC and SIC values.

Then the residual diagnostic was carried out. After analysing the ACF and PACF plots it can be observed that there is serial correlation present at the residuals. The normality test gave the result that the residuals are normally distributed.

It proved that ARCH effect is present at the data set from the heteroscedasticity test. The volatility has been modelled using both ARCH and GARCH models. After analysing different ARCH and GARCH models, GARCH (1, 1) model was chosen as the best model with minimum AIC.

The empirical analysis indicated that the ARIMA (1, 1, 0) + GARCH (1, 1) model provides the optimal results and improves estimation and forecasting the monthly OPEC reference basket crude oil prices.

Therefore, it can be concluded that the combination of ARIMA and GARCH models in handling volatility and the risk return of oil price, made hybrid models to be the most suitable for analysis and forecasting of time series.

Supply and demand of crude oil, impact of natural disasters, gold price are some of the factors that are affecting for the fluctuations of the crude oil price. Sincethe summary measures of the forecasted model show a little high error, as a remedial action multivariate analysis techniques, machine learning techniques etc. can be used as a future study.

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