A Stochastic Model for Variation of Exchange Rates in Sri Lanka

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Abstract *- In Sri Lankan economy, exchange rates have been a one of the critical factors as domestic inflation mostly occurs on depreciation of currency value. Exchange rate is the rate at which currency of a country is bought and sold against the currency of another country in the foreign exchange market. In this work, buying price of the US dollar has been used as a better yardstick to analyze the exchange rates in Sri Lanka as US dollar is being concerned as a world most accepted trade denominator and its buying price was used as purchasing power of the Sri Lankan rupee is deteriorating. The main objectives of this analysis are to derive a stochastic model to represent the variation of exchange rates and hence make conclusions about the variation. Vasicek model was used to model the variation of exchange rates as variation of exchange rates is a stochastic process and there is a mean reversion. Vasicek model is the first stochastic model to capture the value of mean reversion. The model describes the dynamics of the short rate in a linear equation and short rates can be solved explicitly. In this analysis, using daily authentic data from 2005 to 2012 and maximum likelihood estimation, parameters of the Vasicek model were derived. According to the final model, exchange rate is fluctuating around the value Rs.108.3488 and there is an ascent of the variation. Therefore, it is clear that currency value is deteriorating and hence, investors have to pay more money to import their products. Then this additional cost has to be covered by enhancing price levels inside the country. In order to control the depreciation of currency value government intervention is needed. Improvement of the gross domestic product can be concerned as a best economic strategy to regulate the depreciation of currency value.*

Keywords: *stochastic model, exchange rates*

I. INTRODUCTION

In Sri Lankan economy, exchange rates have been a one of the critical factors as domestic inflation mostly occurs on depreciation of currency value. In this work, main objectives are to derive a stochastic model to represent the variation of exchange rates and hence make conclusions about the variation. The exchange rate is the price of a currency in terms of another currency. More precisely, exchange rate is the rate at which currency of a country is bought and sold against the currency of another country in the foreign exchange market[4]. Since US dollar is being concerned as a world most accepted trade denominator, it can be used as a better yardstick to analyze the exchange rates in Sri Lanka. Moreover, buying price of the US dollar is to be used to analyze the exchange rate as purchasing power of the Sri Lankan rupee is deteriorating and hence domestic inflation occurs.

Fluctuation of the exchange rate has been a stochastic process and there is a mean reversion of the variation. Therefore, *Vasicek* model can be used to model the variation of exchange rates in Sri Lanka. *Vasicek* model is the first economic model to capture the value of mean reversion. The model describes the dynamics of the short rate in a linear equation and short rates can be solved explicitly. In *Vasicek* model, the instantaneous exchange rate (r) compiles with the following stochastic differential equation (Rogemor 2004, Yolcu 2005).

$$dr = -\theta(r-\mu)dt + \sigma dw \; ; r(0) = r_0 \text{ , where}$$

$\theta =$ Mean reversion speed

$\mu =$ Long term mean/ mean reversion parameter

$\sigma =$ Standard deviation that determines the volatility of the rate of exchange

$w =$ Wiener process that models the risk factor of random market

II. METHODOLOGY

Probability density function for *Vasicek* model can be derived as follows:

Consider $dr = -\theta(r-\mu)dt + \sigma dw \; ; r(0) = r_0$.
Let $X = r - \mu$.
Then *Vasicek* model can be rewritten as $dX = -\theta X + \sigma dw$.
Define $y(t) = X(t)e^{\theta t}$.
Hence, we will get $dy = \sigma e^{\theta t} dw$ and this implies $y(t) = y(0) + \sigma \int_0^t e^{\sigma\tau} dw(\tau)$.
Therefore $y(t) \sim Normal\left(X_0, \sigma^2 \int_0^t e^{2\theta\tau}\, d\tau\right)$ as $dw \sim Normal(0, d\tau)$.
This implies $X(t) \sim Normal\left(X_0 e^{-\theta t}, \sigma^2 e^{-2\theta t} \int_0^t e^{2\theta\tau} d\tau\right)$.
Therefore we have $r(t) \sim Normal\left\{\mu + (r_0 - \mu)e^{-\theta t}, \sigma^2 e^{-2\theta t} \int_0^t e^{2\theta\tau} d\tau\right\}$.
i.e. $r(t) \sim Normal\left[\mu + (r_0 - \mu)e^{-\theta t}, \frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta t}\right)\right]$.
Then the probability density function of *Vasicek* model is given by

$$p(r) = \left[\frac{\pi\sigma^2}{\theta}\left(1 - e^{-2\theta t}\right)\right]^{\frac{-1}{2}} exp\left\{-\frac{\theta\left[r - \mu - (r_0 - \mu)e^{-\theta t}\right]^2}{\sigma^2\left(1 - e^{-2\theta t}\right)}\right\}.$$

Then the Likelihood function is

$$L\left(\{\theta, \mu, \sigma^2\} \middle| r_0, r_1, \dots, r_{n-1}\right) = p\left(r_0, r_1, \dots, r_{n-1} \middle| \{\theta, \mu, \sigma^2\}\right) = \prod_{i=0}^{n-1} p(r_i | \{\theta, \mu, \sigma^2\})$$

Now define $L' = \frac{1}{n}\, lnL$.

Then $L' = \frac{1}{n}\sum_{i=0}^{n-1} ln\, p(r_i | \{\theta, \mu, \sigma^2\}) = \left(\frac{-1}{2}\right)\sum_{i=0}^{n-1} ln\left[\frac{\pi\sigma^2}{\theta}\left(1 - e^{-2\theta t}\right)\right]\left\{-\frac{\theta\left[r_i - \mu - (r_0 - \mu)e^{-\theta t}\right]^2}{\sigma^2\left(1 - e^{-2\theta t}\right)}\right\}$.

It is natural to estimate the following funtions of the estimators:

$$\alpha := e^{-\theta t}, \qquad \beta := \mu, \qquad v^2 := \frac{\sigma^2}{2\theta}\left(1 - e^{-2\theta t}\right)$$

Now consider the equations $\frac{\partial L'}{\partial \alpha} = 0, \frac{\partial L'}{\partial \beta} = 0, \frac{\partial L'}{\partial v} = 0$ and evaluating these partial differential equations at $\alpha = \alpha', \beta = \beta'$ and $v = v'$ respectively, maximum likelihood estimators (MLE) for $\alpha, \beta,$ and v^2 can be obtained as follows (Brigo 2nd ed.):

$$\alpha' = \frac{n\sum_{i=1}^{n} r_i r_{i-1} - \sum_{i=1}^{n} r_i \sum_{i=1}^{n} r_{i-1}}{n\sum_{i=1}^{n} r_i^{\,2} - \left(\sum_{i=1}^{n} r_{i-1}\right)^2}$$

$$\beta' = \frac{\sum_{i=1}^{n}\left[r_i - \alpha' r_{i-1}\right]}{n(1 - \alpha')}$$

$$v'^2 = \frac{1}{n}\sum_{i=1}^{n}\left[r_i - \alpha' r_{i-1} - \beta'(1 - \alpha')\right]^2$$

In order to calculate α', β' and v'^2, daily buying price of the US dollar from 2005 to 2012 can be taken directly from Central Bank of Sri Lanka and hence stochastic model for exchange rate is to be derived.

III. RESULTS AND DISCUSSION

MLE values of the data set can be computed as $\theta = 0.0519$, $\mu = 108.3488$ and $\sigma = 0.3340$.
Hence, stochastic model for exchange rates in Sri Lanka can be derived as

$$dr = -0.0519(r - 108.3488)dt + 0.3340\ dw.$$

But it is well know that $dw = \in \sqrt{dt}\ ;\ \in \sim Normal(0,1)$ (Hull, 7th ed)[4].
Then this leads to

$$dr = -0.0519(r - 108.3488)dt + 0.3340 \in \sqrt{dt}\ ; \in \sim Normal(0,1).$$

Therefore, according to this stochastic model, exchange rate is fluctuating around the value Rs.108.3488 and there is an ascent of the variation. Moreover, this model can be used to forecast the exchange rate in Sri Lanka. Variation of actual exchange rate and simulated exchange rate are shown in Figure 1.

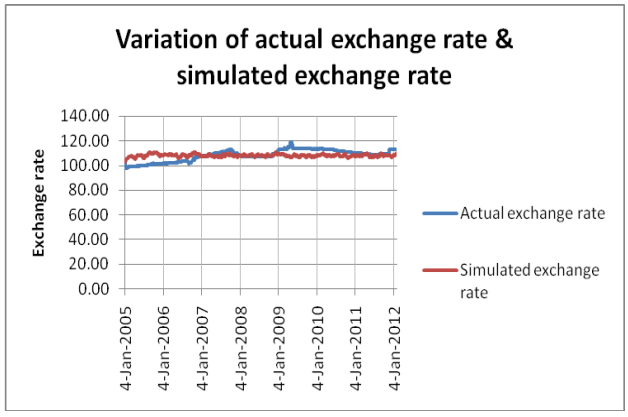

Figure 1

According to this figure, for some time periods, there is a gap between actual data and simulated data. This error can be minimized by running a Monte Carlo simulation. Deterministic part of the final stochastic differential equation is goinig to be $dr = -0.0519(r - 108.3488)t$ and hence deterministic process is going to be $r(t) = 108.3488 - 5.7088\ exp(-0.0519t)$.
Therefore, it can be strongly concluded that exchange rate attains to the value Rs.108.3488 for long time periods.

IV. CONCLUSIONS/RECOMMENDATIONS

In Sri Lanka, it is clear that currency value is deteriorating in relation to the currencies of other countries. Therefore investors have to pay more money to imports their products. Then this additional cost has to be covered by increasing the price levels inside the country. As a general rule, a country with a consistently lower inflation rate exhibits a rising currency value, as its purchasing power increases relative to other currencies. In order to control the depreciation of currency value in Sri Lanka government intervention is needed. Improvements of the gross domestic product (GDP) can be concerned as a best economic solution to regulate the depreciation of currency value.

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