Mathematics of architecture: A study of the philosophy of emerging architecture

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Abstract— *Mathematics has been used in architectural design since the ancient times. Modern architectural forms designed on digital design platforms based on complex mathematical equations differ from Euclidean geometry. The non-Euclidean mathematical concepts behind modern architectural forms were studied within the scope of this research.*

All of nature's work has a mathematical logic. After identifying and observing the mathematical principles in nature, a simple design of a student seating area was carried out incorporating the mathematical concepts used by nature in its design. This research explores the possibility of utilizing mathematics in nature to design efficient building forms.

I. Introduction

Architecture and mathematics are two distinct human endeavors. Yet both disciplines are very closely related having their deepest roots embedded in geometry. Mathematics is an integral component of architecture and the two disciplines have up to the present time maintained close connections. One of the main areas of research was to explore the relationship between architecture and mathematics and to discover the close connection maintained between architecture and mathematics from the ancient times up to the present day.

The link between architecture and mathematics goes back to the ancient times where the two disciplines were virtually indistinguishable. The first use of mathematics in architecture started early with the Egyptians over 4600 years ago with the design of the pyramids. The enduring beauty of ancient architecture such as The Parthenon, the Taj Mahal, The Notre Dame and many more other architectural wonders was a result of the paradoxical combination between architecture and mathematics. Within the scope of this research an attempt was made to identify the mathematical principles governing the design of ancient architectural forms.

Today mathematics continue to feature prominently in building design and mathematics is more integral to architecture than ever before. The power and complexity of mathematical processes in architecture is growing. Access to electronic computational power in tandem with accessible graphical interaction has given architects new creative opportunities with which to access geometrical space. As a result architects can now explore a variety of exciting design options based on complex mathematical calculations allowing them to build ground breaking forms. A main area of research was dedicated to discovering the non-Euclidean mathematical principals behind modern architectural forms and to understand how architects have incorporated mathematical algorithms into their designs.

There exists in nature many phenomena that can be described in the language of mathematics. All of nature's work have a mathematical logic. One of the main objectives of this research was to discover the mathematical principals in nature and then to understand how this knowledge could be used to improve the appearance and function of architecture.

II. Mathematical Principles in Architecture

A. Euclidean and Non-Euclidean Geometry

The geometry of Euclid is a model of deductive reasoning. Euclidean Geometry is the study of flat space. It includes the parallel postulate which states that through a point P not on a line L, there is only one line through P that is parallel to L.

The parallel postulate came into great debate among mathematicians in the late 19th century and it was discovered that the geometry of Euclid fails to describe all physical space. Any form of geometry that contains a postulate that challenges Euclid's parallel postulate is called non Euclidean Geometry.

Riemann Geometry is a form of non-Euclidean Geometry that contains a parallel postulate equivalent to "If L is any line and if P is any point not on L, then there are no lines through P parallel to L" (Miller, Mathematical Ideas). Riemann Geometry is the study of curved surfaces.

Hyperbolic Geometry is a form of non-Euclidean geometry that contains a parallel postulate equivalent to " If L is any line and if P is any point not on L, then there exists at least two lines through P parallel to L" (Miller, Mathematical Ideas). Hyperbolic Geometry, also known as Saddle Geometry, is the study of saddle shaped space.

To the ancient Greeks, mathematics had meant geometry above all and the Greek view of geometry was the geometry of Euclid. This is evident through ancient Greek architecture such as in the Parthenon. The influence of Euclidean geometry on architecture can also be seen in ancient European castles.

Figure 1: Euclidean architecture

The concept of non-Euclidean geometry has freed the architect from the static Cartesian stricture of the two dimensional drafting plane resulting in ground breaking architectural forms. The Cybertecture Egg in Mumbai India and the Chinese National Stadium in Beijing are a few examples of non-Euclidean architectural forms.

Figure 2: The Cybertecture Egg, Mumbai, India

Figure 3: The Birds Nest, Beijing, China

B. Extrinsic and Intrinsic Curvature

The Westhafen Tower in Frankfurt Germany designed by the architects Schneider & Schumacher exhibits extrinsic curvature.

Figure 4: Westhafen tower, Frankfurt, Germany

Extrinsic properties are properties of a curve or surface that depends on the coordinate space they are embedded in. Extrinsic curvature arises when a surface curves into a higher dimension. It is a type of single curvature meaning it is curved only on one linear axis. This type of curvature is not detectable to the inhabitants of the surface, it can only be identified by those who study the three dimensional space surrounding the surface.

The roof structure of the Denver International Airport exhibits intrinsic curvature. Intrinsic properties of a surface are those that can be measured within the surface itself without referring to any larger space.

Figure 5: Roof structure of the Denver International Airport

A surface exhibits intrinsic curvature when the geometry within the surface differs from flat Euclidean geometry. Intrinsic curvature is detectable to the inhabitants of the surface, not just the outside observers. Intrinsic curvature is a type of double curvature meaning it is curved on two linear axes.

C. Minimal Surfaces

Surfaces with zero mean curvature are called minimal surfaces because they minimize the surface area. Such a surface has a minimal surface area for given boundary conditions. In other words a minimal surface is one where the sum of the curvatures in two principal directions on the surface is zero.

"Minimal surfaces, including those expressed by a soap film and their use as models for geometry and architecture, constitute a specific aspect of the relations between architecture and mathematics" (Emmer, 2012)

Architectural forms based on the concept of minimal surfaces not only utilize a minimum amount of space but such structures are also stable since they contain low energy. The proposal for the RTV Headquarters in Zurich by Oliver Dibrova is an example on the use of minimal surfaces in architectural design.

Figure 6: Proposal for the RTV headquarters, Zurich

D. Non-orientable Surfaces

The proposed Arts and Literature Centre for the Taiwanese city of Taichung, the “Swallow’s Nest” by Vincent Callebaut is a non-orientable surface.

Figure 7: Swallows Nest, Taichung Taiwan

A surface is termed as orientable if it consists of two sides. Non-orientable surfaces have only one side. A non-orientable surface is a one sided surface which if travelled upon could be followed back to the point of origin while flipping the traveler upside down. The Mobius strip and the Klein Bottle are examples of a non-orientable surfaces. A surface is considered to be non-orientable if and only if a Mobius band can be found within that surface.

E. Modular Architecture

Modular architecture refers to the design of a system composed of separate components that can be connected together. The beauty of modular architecture is that any one component can be added or replaced without affecting the rest of the system.

The Nakagin Capsule Tower designed by the architect Kisho Kurokawa in Shimbashi Tokyo Japan embodies the principal of modular architecture. The building is composed of 140 prefabricated capsules which are attached independently and is cantilevered from the shaft so that any capsule may be removed easily without affecting the other capsules

Figure 8: Nakagin Capsule Tower, Tokyo Japan

F. Ruled Surfaces

A surface S is considered as a ruled surface if through point on S there is a straight line that lies on S. A surface S is termed as a doubly ruled surface if through every point on S there are two distinct straight lines that lies on S. Hyperboloid of one sheet and a Hyperbolic Paraboloid can be considered as doubly ruled surfaces.

The roof of the school at Sagrada Familia is a sinusoidal ruled surface and the roof of the Warszawa Ochota Railway Station in Warsaw Poland is a hyperbolic paraboloid.

Figure 9: Roof of the school at Sagrada Familia

Figure 10: Roof of the Warszawa Ochota railway station, Warsaw Poland

G. Chaos and Geometric Order

In architectural composition geometric order and chaos are basic components.

Geometric order is represented by ideal mathematical forms and ideal relationships. The application of geometric order to architecture has been practiced since the ancient times to confer the importance or monumentality of a structure. The pyramids, gothic cathedrals and the Doric temple are few architectural structures based on geometric order. Ieoh Ming Pei’s glass pyramid which is the modern entrance to the Louvre Museum in Paris embodies the concept of geometric order.

Figure 11: Entrance to the Louvre Museum, Paris

Chaos is the complete opposite of geometric order. It is represented by forms and relationships that are complex and difficult to be explained using Euclidean geometry. The UFA Cinema Centre in Dresden is an example of chaos in architecture as it makes the impression of random composition of different components.

Figure 12: 1UFA Cinema Centre, Dresden

But chaos and geometric order are strongly connected. “Elimination of chaos from architectural composition creates spatial boredom. Elimination of geometric order causes ill eligibility” - (Rubinowicz, 2000)

III. MATHEMATICAL PRINCIPLES IN NATURE

A. *Fractal Geometry*

A fractal is a mathematical recursive pattern with the smaller parts mirroring the larger parts. Even though fractals are a mathematical construct they are found ubiquitously in each major realm of nature. The patterns in the Romanesco broccoli, branching in a tree, leaf venation, flowering in a dill plant and the arrangement of fronds in a fern all follow the concept of fractal geometry.

Figure 13: Romanesco broccoli

Figure 14: Flowering in a dill plant

Architect Xavier Vilalta had incorporated the fractal geometry utilized by nature, into the design of an unconventional shopping mall called Lideta Mercato in Addis Ababa in Ethiopia. The shopping mall had been designed based on the fractal geometry of a Menger sponge. It consists of concrete blocks with square cut outs, that is used to create a textile like façade and to provide natural lighting and ventilation. This is an example utilizing nature’s design principles to improve the form and function of the architecture.

Figure 15: Lideta Mercato, Ethiopia

B. *Golden Ratio and the Fibonacci Sequence*

The golden ratio and the Fibonacci Sequence appears in many patterns in nature including the spiral arrangement of leaves, branching of trees, in the fruit sprouts of a pineapple, arrangement of seeds in a sunflower etc.

Plants grow new cells in spirals. To obtain maximum packing in a horizontal space with no gaps the turn must be equal to the golden ratio. In most instances the number of spirals in the clockwise and anticlockwise directions are two consecutive Fibonacci numbers.

The use of the golden ratio can be seen in the design of the largest of the pyramids in Giza. When a right angled triangle is created using the basic Phi relationships it forms the dimensions of the great pyramid of Giza.

Phi has the mathematical property of its square being one more than itself.

$$\varphi + 1 = \varphi^2$$

By applying the Pythagoras theorem to the above equation a right angled triangle with sides $\sqrt{\varphi}$, 1 and ϕ can be obtained. This right angled triangle is known as the Golden Triangle. When two golden triangles placed back to back it will form a golden pyramid.

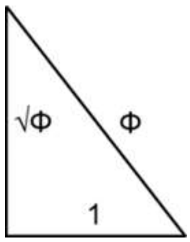

Figure 16: The golden triangle

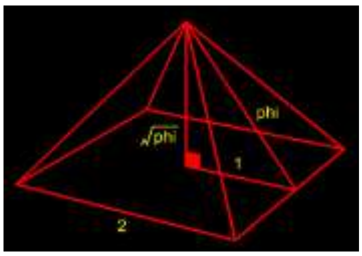

Figure 17: The golden pyramid

Then the ratio of the height of the pyramid to the base of the pyramid would be;

$$\frac{Height\ of\ the\ Pyramid}{Width\ of\ the\ Pyramid} = \frac{1.272}{2} = 0.636$$

The width of the great pyramid is approximately 230.4 m and the estimated original height is 146.5 m.

$$\frac{Height\ of\ the\ Pyramid}{Width\ of\ the\ Pyramid} = \frac{230.4}{146.5} = 0.636$$

This indicates that the great pyramid of Giza consists of golden triangles.
The design of the UN Headquarters employs the golden ratio in many ways. The application of the golden ratio is most evident when considering the width of the entire building and comparing it to the height of every 10 floors. The rectangles thus formed are all golden rectangles.

Figure 18: Golden rectangles enclosing the UN headquarters building

The building is designed with four non-reflective bands, located with 5, 9, 11 and 10 floors between them. This configuration divides the building at several golden ratio points. The building is approximately 41 floors tall. Based on the height of the building it can be shown that the first golden ratio point midway between the 15th and the 16th floors (illustrated using red lines in Figure 19).

$$\frac{41\ Floors}{1.618^2} = 15.7\ Floors$$

Therefore the golden ratio point defines the middle of the 2nd non-reflective band.
Based on the distance from the top of the building to the middle of the 1st non-reflective band it can be shown that the second golden ratio point lies midway between the 26th and the 27th floor (illustrated using orange lines in Figure 19).

$$\frac{(41-5.5)\ floors}{1.618^2} = 21.9\ floors\ from\ the\ top$$

Therefore the 3rd non-reflective band is located at the second golden ratio point.
Based on the distance from the base of the building to the top of the 2nd non-reflective band it can be shown that the third golden ratio point lies on the 6th floor (illustrated using purple lines in Figure 19)

$$\frac{16\ Floors}{1.618} = 6.1\ Floors$$

This defines the position of the 1st and 2nd non-reflective bands.

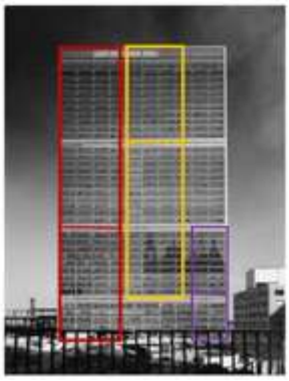

Figure 19: Golden ratio in the UN headquarters building

IV. DESIGN OF THE STRUCTURE

The venation pattern of the Frangipani leaf was traced in Rhino 3D

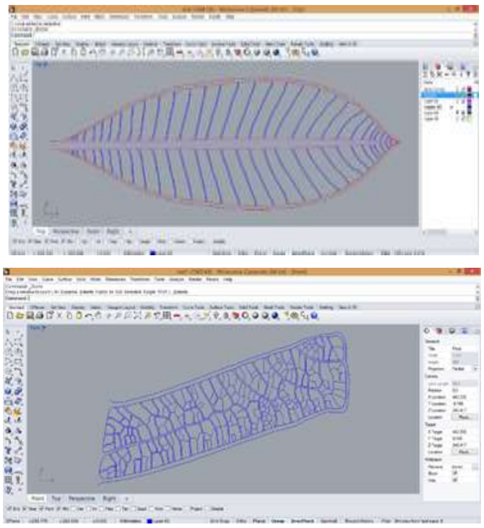

Figure 20: The venation pattern of the Frangipani leaf

By closely observing the venation pattern an algorithm encompassing its design was discovered. The venation consisted of fractals. From the straight edge of the main leaf numerous rectangular like sections are born. From the straight edges of those rectangular sections smaller rectangular like sections are formed. This algorithm is repeated indefinitely to give the structural pattern of the leaf.

A student seating area was designed using the algorithm used by nature in the design of the Frangipani leaf. The leaf venation pattern was traced onto a square of size 20m x 20m and then the four edges of the square were bent at 90º to the original surface.

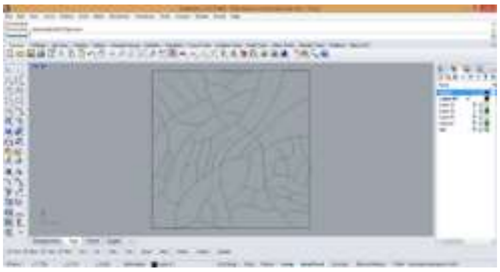

Figure 21: Leaf venation pattern traced into a square

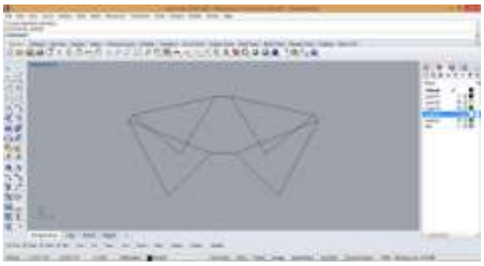

Figure 22: The edges of the square bent at 90º

Thus a structure supported solely on the fractal venation of the leaf was obtained.

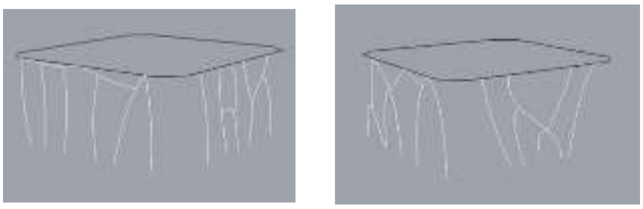

Figure 23: Side walls of the structure

Figure 24: Top view of the structure

The structure was designed with 2mm steel plates and glass plates.

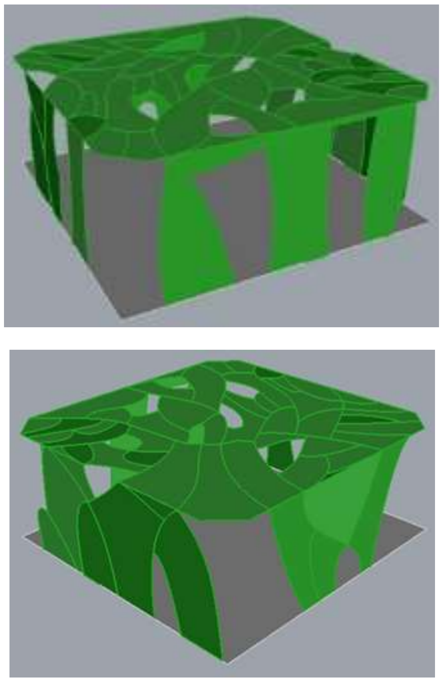

Figure 25: The designed structure

V. CONCLUSION

Mathematics is an integral component of architecture. Architecture relies on mathematics to achieve visual harmony, structural integrity and logical construction. However modern and different new architecture is mathematics is still at its heart.

Mathematics is the blueprint of nature's designs. Natural structures that stand the test of time can provide the same engineering stability and architectural sophistication when replicated.

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